

Mean Absolute Deviation



Getting the Idea

Another way to measure the variability of a data set is to measure variation from the mean. You do this by measuring how far each individual value is from the mean.

To measure variability away from the mean, first find the mean of the data set. Next, find the absolute value of the difference between the mean and each value of the data set. This gives the deviation of each value from the mean. Then find the sum of all the deviations and divide the sum by the number of values in the data set. The average of the absolute deviations from the mean is called the **mean absolute deviation**.

Suppose the heights, in inches, of three plants, are: 18, 27, and 21.

Find the mean of the heights, in inches.

$$\text{mean} = \frac{18 + 27 + 21}{3} = \frac{66}{3} = 22$$

Find how each height differs from the mean height. This is the deviation from the mean.

$$\text{deviation of first value} = 18 - 22 = -4$$

$$\text{deviation of second value} = 27 - 22 = 5$$

$$\text{deviation of third value} = 21 - 22 = -1$$

Notice that the average deviation of the values will be zero.

So, take the absolute value of each deviation.

$$|-4| = 4$$

$$|5| = 5$$

$$|-1| = 1$$

Now find the average of the absolute deviations.

$$\text{mean absolute deviation} = \frac{4 + 5 + 1}{3} = \frac{10}{3} = 3.\bar{3}$$

So, the plant heights vary by an average of $3.\bar{3}$ inches from the mean.

When the mean absolute deviation is small, it means the data is bunched closely together. For the plant heights, the mean absolute deviation is $3.\bar{3}$, which is relatively small. This makes sense since the plant heights are not very different. So, there is not much variability in the plant heights.

If the mean absolute deviation is large, it means the data is spread out and has greater variability.

Example 1

Find the mean absolute deviation for the following quiz scores: 6, 9, 6, 9, 8, and 10. The mean score on the quizzes is 8.

Strategy Find the deviation of each score from the mean score. Then find the absolute deviations to get the mean absolute deviation.

Step 1

Find the deviation of each score from the mean score.

Subtract each score from the mean score to find the deviations.

$$6 - 8 = -2$$

$$9 - 8 = 1$$

$$6 - 8 = -2$$

$$9 - 8 = 1$$

$$8 - 8 = 0$$

$$10 - 8 = 2$$

Step 2

Find the absolute deviations, the absolute value of each deviation.

$$|-2| = 2$$

$$|1| = 1$$

$$|-2| = 2$$

$$|1| = 1$$

$$|0| = 0$$

$$|2| = 2$$

Step 3

Find the mean absolute deviation.

$$\text{Add the absolute deviations: } 2 + 1 + 2 + 1 + 0 + 2 = 8$$

There are 6 quiz scores in the set.

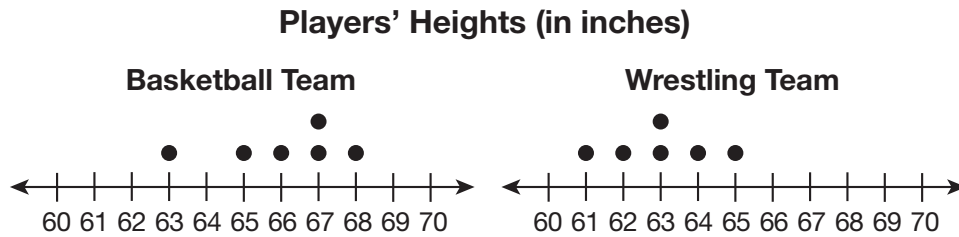
$$\text{mean absolute deviation} = \frac{8}{6} = 1.\bar{3}$$

Solution The mean absolute deviation is $1.\bar{3}$.

You can use the mean absolute deviation to compare two populations.

Example 2

The heights of players on two school teams are shown on the dot plots below.



Compare the variability in the mean heights of the players on each team.

Strategy Find the mean absolute deviation of each team's heights.

Step 1

Find the mean absolute deviation of the basketball team's heights.

$$\text{mean} = \frac{63 + 65 + 66 + 67 + 67 + 68}{6} = \frac{396}{6} = 66$$

Find the absolute deviation of each height from the mean height.

$$|63 - 66| = |-3| = 3 \qquad |67 - 66| = |1| = 1$$

$$|65 - 66| = |-1| = 1 \qquad |67 - 66| = |1| = 1$$

$$|66 - 66| = |0| = 0 \qquad |68 - 66| = |2| = 2$$

Find the mean absolute deviation.

$$\text{mean absolute deviation} = \frac{3 + 1 + 0 + 1 + 1 + 2}{6} = \frac{8}{6} = 1.\bar{3}$$

Step 2

Find the mean absolute deviation of the wrestling team's heights.

$$\text{mean} = \frac{61 + 62 + 63 + 63 + 64 + 65}{6} = \frac{378}{6} = 63$$

Find the absolute deviation of each height from the mean height.

$$|61 - 63| = |-2| = 2 \qquad |63 - 63| = |0| = 0$$

$$|62 - 63| = |-1| = 1 \qquad |64 - 63| = |1| = 1$$

$$|63 - 63| = |0| = 0 \qquad |65 - 63| = |2| = 2$$

Find the mean absolute deviation.

$$\text{mean absolute deviation} = \frac{2 + 1 + 0 + 0 + 1 + 2}{6} = \frac{6}{6} = 1$$

Step 3

Compare the mean absolute deviations.

$$1.\bar{3} > 1$$

The mean absolute deviation of the basketball team's heights is slightly more than the mean absolute deviation of the wrestling team's heights.

This corresponds to the dot plots. The dot plot for the basketball team shows somewhat more variability than the dot plot for the wrestling team.

Solution There is a little more variability in the heights of the basketball team than in the heights of the wrestling team.

Example 3

The numbers of pages in books read by sixth- and seventh-grade students during one semester are shown below.

Sixth grade: 125, 132, 150, 137

Seventh grade: 198, 174, 208, 120

Compare the variability in the mean number of pages read by students in each grade.

Strategy Find the mean absolute deviations of the pages read by each grade.

Step 1

Find the mean absolute deviation of the pages read by sixth graders.

$$\text{mean} = \frac{125 + 132 + 150 + 137}{4} = \frac{544}{4} = 136$$

Find the absolute deviations.

$$|125 - 136| = |-11| = 11 \quad |150 - 136| = |14| = 14$$

$$|132 - 136| = |-4| = 4 \quad |137 - 136| = |1| = 1$$

Find the mean absolute deviation.

$$\text{mean absolute deviation} = \frac{11 + 4 + 14 + 1}{4} = \frac{30}{4} = 7.5$$

Step 2

Find the mean absolute deviation of the pages read by seventh graders.

$$\text{mean} = \frac{198 + 174 + 208 + 120}{4} = \frac{700}{4} = 175$$

Find the absolute deviations.

$$|198 - 175| = |23| = 23 \quad |208 - 175| = |33| = 33$$

$$|174 - 175| = |-1| = 1 \quad |120 - 175| = |-55| = 55$$

Find the mean absolute deviation.

$$\text{mean absolute deviation} = \frac{23 + 1 + 33 + 55}{4} = \frac{112}{4} = 28$$

Step 3

Compare the mean absolute deviations.

$$7.5 < 28$$

The mean absolute deviation in the number of pages read by the sixth graders is much less than the mean absolute deviation for the seventh graders. 28 is almost 4 times 7.5.

Solution The variability in the number of pages read by the seventh graders is almost 4 times the variability in the number of pages read by the sixth graders.



Coached Example

The weights, in pounds, of the dogs that boarded at a veterinarian's clinic over the weekend were: 43, 87, 12, 15, and 23.

Find the mean absolute deviation of the weights of the dogs that boarded at the clinic.

Find the mean weight of the dogs that boarded at the clinic.

To find the deviations, _____ each weight from the mean weight.

$$43 - \underline{\quad} = \underline{\quad} \quad 87 - \underline{\quad} = \underline{\quad} \quad 12 - \underline{\quad} = \underline{\quad}$$

$$15 - \underline{\quad} = \underline{\quad} \quad 23 - \underline{\quad} = \underline{\quad}$$

To find the absolute deviations, find the _____ of each deviation.

$$|\underline{\quad}| = \underline{\quad} \quad |\underline{\quad}| = \underline{\quad} \quad |\underline{\quad}| = \underline{\quad}$$

$$|\underline{\quad}| = \underline{\quad} \quad |\underline{\quad}| = \underline{\quad}$$

Add the absolute deviations.

Find the average of the absolute deviations.

$$\text{mean absolute deviation} = \underline{\quad} = \underline{\quad}$$

The mean absolute deviation of the weights of the dogs is _____ pounds.



Lesson Practice

Choose the correct answer.

Use the following information for questions 1 and 2.

Paula's grades on her history tests this semester are 79, 93, 92, 86, and 90.

- Which shows the deviation of each of her grades from her mean grade?
 - $-9, 5, 4, -2, 2$
 - $-8, 6, 3, -3, 2$
 - $-11, 6, 5, -4, 4$
 - $-14, 9, 5, -3, 2$
- What is the mean absolute deviation of Paula's history grades?
 - 0
 - 4.2
 - 4.4
 - 22

Use the following information for questions 3 through 5.

The lengths, in seconds, of four folk songs are 128, 165, 182, and 141.

The lengths, in seconds, of four pop songs are 90, 98, 102, and 94.

- What is the mean absolute deviation, in seconds, of the folk songs?

A. 18	C. 19.5
B. 18.25	D. 19.75
- What is the mean absolute deviation, in seconds, of the pop songs?

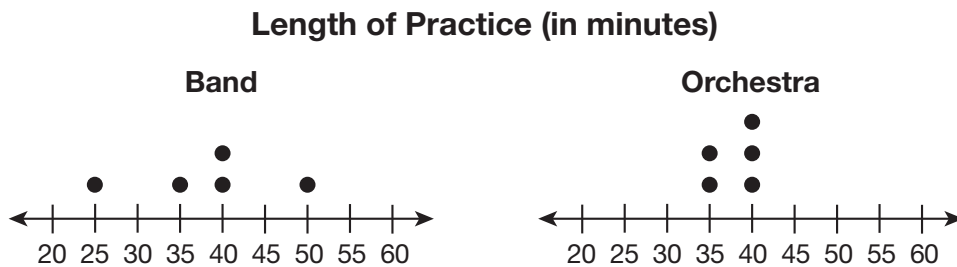
A. 2	C. 6
B. 4	D. 8
- Which of the following statements is true?
 - The variability in the times of the folk songs is about half that of the pop songs.
 - The variability in the times of the folk songs is about twice that of the pop songs.
 - The variability in the times of the folk songs is about 3 times that of the pop songs.
 - The variability in the times of the folk songs is about 4 times that of the pop songs.

Use the following information for questions 6 and 7.

The ages of volunteers at a hydration station during a marathon are 16, 27, 31, 24, 19, and 63. The ages of volunteers at the finish line during a marathon are 32, 28, 25, 20, 40, and 35.

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|--|--|
| <p>6. What is the mean absolute deviation, in years, of the ages of the volunteers at the hydration station?</p> <p>A. $11.\bar{3}$</p> <p>B. $11.\bar{6}$</p> <p>C. $12.\bar{3}$</p> <p>D. $12.\bar{6}$</p> | <p>7. What is the mean absolute deviation, in years, of the ages of the volunteers at the finish line?</p> <p>A. $4.\bar{3}$</p> <p>B. $5.\bar{6}$</p> <p>C. $6.\bar{3}$</p> <p>D. $6.\bar{6}$</p> |
|--|--|

8. The lengths, in minutes, of the school band and orchestra practices are shown on the dot plots below.

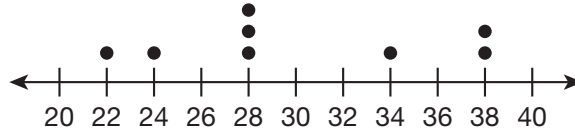


- A. What is the mean absolute deviation, in minutes, of the length of each group's practice? Show your work.

- B. How does the variability in the length of band practice compare to the variability in the length of orchestra practice? Explain your thinking.

9. The dot plot shows the distances jumped at the long jump competition in the city championships. Find each deviation from the mean. Write each number in the correct box.

Distances Jumped (in inches)

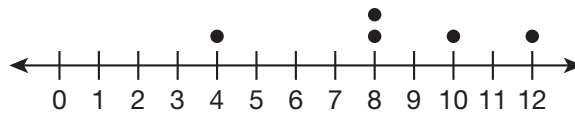


-8	-6	-5	-2	2	4	8	10
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Deviation from Mean	Not a Deviation from Mean

10. Tim kept track of the number of points he scored in each game during a tournament. Circle the absolute deviation for each data point.

Points Scored



- A. 1.6
- B. 3.4
- C. 4.4
- D. 6.6
- E. 1.4
- F. 0.4
- G. 3.6

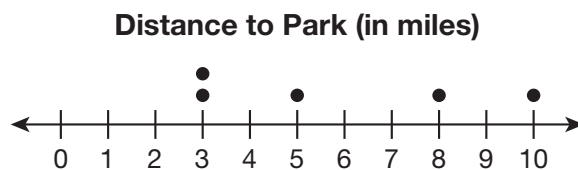
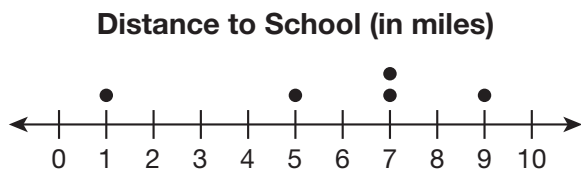
11. The ages of the members of the Senior Dance Team are shown below. Circle the mean absolute deviation.

13, 14, 14, 15, 16, 16, 17, 17, 18, 18

mean absolute deviation =

1.44
7.89
15.8
14.4

12. Five friends determined the distances, in miles, from each of their houses to the school and to the park. Is each statement true? Select Yes or No.



- A. The means of both sets of data are equal. Yes No
- B. The mean absolute deviation is greater for the distances to the park than the distances to the school. Yes No
- C. The mean absolute deviation for distances to the park is 2.56 miles. Yes No
- D. The absolute deviations of the distances to the school are 5.8, 1.16, 0.83, 0.83, and 0.64. Yes No